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Study of the nonlocal gauge invariant mass operator $\text{Tr} \int d^4x F_{\mu\nu} (D^2)^{-1} F_{\mu\nu}$ in the maximal Abelian gauge

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Abstract

The nonlocal gauge invariant mass operator $\text{Tr} \int d^4x F_{\mu\nu} (D^2)^{-1} F_{\mu\nu}$ is investigated in Yang–Mills theories in the maximal Abelian gauge. By means of the introduction of auxiliary fields a local action is achieved, enabling us to use the algebraic renormalization in order to prove the renormalizability of the resulting local model to all orders of perturbation theory.

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1. Introduction

One of the major open problems in quantum field theory is the understanding of non-Abelian gauge theories, and consequently of quantum chromodynamics (QCD), in the infrared regime. The confinement phenomenon of quarks and gluons is not yet clearly established from the theoretical point of view and still waits for a satisfactory explanation.

The Yang–Mills (YM) theories are described by the following Euclidean action:

$$S_{\text{YM}} = \frac{1}{4} \int d^4x F_{\mu\nu}^A F_{\mu\nu}^A, \quad (1)$$

where $F_{\mu\nu}^A$ is the field strength

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + g f^{ABC} A_\mu^B A_\nu^C. \quad (2)$$

Here f^{ABC} are the structure constants of the gauge group $SU(N)$ with $A = 1, \dots, N^2 - 1$, and g is the coupling constant. At high energies, the running coupling constant is sufficiently

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small to allow us for a perturbative description, as expressed by the asymptotic freedom [1, 2]. However, when one lowers the energy, the running coupling constant grows, causing perturbation theory to fail, so that nonperturbative techniques are required.

To deal with this problem, different approaches have been considered. For example, in the Landau gauge, several analytic results have been obtained on the infrared behavior of the propagators of the theory, see for instance [3–11]. In this gauge, lattice simulations have confirmed an infrared suppressed gluon propagator exhibiting violation of positivity [12–16], a feature interpreted as a signal of confinement. In particular, the fitting of the lattice data for the gluon propagator is usually accomplished with the aid of several mass parameters [12, 17, 18], whose theoretical interpretation is still under investigation. So far, two possible origins could be suggested for such massive parameters, namely: the existence of the Gribov copies [3] and the condensation of suitable dimension two operators built up with gluon and ghost fields [19–21]. In this work we shall focus on dimension two operators built up with gluon fields only, see [22, 23] for a recent discussion of ghost condensation in the Landau and maximal Abelian gauges.

The massive Gribov parameter γ , which is fixed by a gap equation, follows from the restriction of the domain of integration in the Feynman path integral to the so-called Gribov region [3–5]. This restriction is needed in order to deal with the problem of the Gribov copies and may be implemented through the introduction in the YM action of a nonlocal term, known as the Zwanziger horizon function [4]. However, there is still room for additional mass parameters. Therefore, the possibility of the condensation of dimension two operators, giving rise to a dynamically generated mass for the gluons, has been taken into account. Let us also mention that, besides the lattice data, the introduction of an effective gluon mass turns out to be useful also from the phenomenological point of view, see for example [24–26]. In particular, in the Landau gauge, the dimension two operator $A_\mu^A A_\mu^A$ was proven to be multiplicatively renormalizable to all orders [27]. In [21, 28–30], an effective potential for $A_\mu^A A_\mu^A$ was constructed. The formation of a nonvanishing condensate $\langle A_\mu^A A_\mu^A \rangle$, resulting in a dynamical effective gluon mass, turned out to be energetically favored.

Besides the Landau gauge, other gauge fixings have been considered. We mention the recent analysis of dimension two operators in the Curci–Ferrari [31] and general linear covariant gauges [32, 33]. In [31], an effective potential was constructed for the on-shell BRST invariant operator $(\frac{1}{2}A_\mu^A A_\mu^A + \alpha \bar{c}^A \bar{c}^A)$ in the Curci–Ferrari gauge while, in [32, 33], a detailed study of the already mentioned operator $A_\mu^A A_\mu^A$ was performed in the linear covariant gauges. As a result, in [33], it was shown that the gluons do acquire a dynamical mass since the formation of $\langle A_\mu^A A_\mu^A \rangle$ is energetically favored. Another interesting gauge which has received increasing attention in the last few years is the maximal Abelian gauge (MAG) [34–36]. Several results have already been established in this gauge, both from theoretical [37–41] and lattice [42–44] points of view. The MAG is well suited for the study of special aspects of infrared QCD and color confinement as, for instance, the dual superconductivity and the so-called Abelian dominance. The dual superconductivity mechanism [45–47] asserts that the low-energy regime of Yang–Mills theories should exhibit monopoles as vacuum configurations. The condensation of these magnetic charges might give rise to a dual Meissner effect in the chromoelectric sector. As for the Abelian dominance hypothesis [48], the infrared limit of QCD should be described by an effective theory constructed only from Abelian degrees of freedom, identified with the diagonal components of the gauge field, corresponding to the generators of the Cartan subgroup of the gauge group. Lattice numerical simulations in the MAG have reported significant differences between the diagonal and off-diagonal components of the gluon propagator [42–44]. In particular, the off-diagonal gluon propagator displays a mass greater than that reported for the diagonal component, corroborating in fact the Abelian

dominance hypothesis. In an attempt to understand those lattice results in the MAG, here also the condensation of dimension two operators has been considered. In [37, 49], a dynamical mass generation mechanism for the off-diagonal gluons was proposed in the MAG, by means of the condensation of the operator $(\frac{1}{2}A_\mu^a A_\mu^a + \alpha \bar{c}^a \bar{c}^a)$.⁴

As the reader may have noticed, the dimension two operators mentioned above are gauge dependent, being related to specific choices of the gauge fixing. This is a consequence of the fact that a *local gauge invariant* dimension two operator is not available in YM theories. Still, the condensation of these operators might be taken as evidence in favor of the existence of a more fundamental *gauge invariant* operator. However, willing to preserve gauge invariance, we are led to give up the locality requirement. The price one has to pay for that is that nonlocal operators are difficult to be handled within a consistent renormalizable framework. So far, several possibilities have been considered. The first proposal for a condensate of dimension two was made by [19, 20], who considered the nonlocal gauge invariant operator A_{\min}^2 , obtained by minimizing the operator $A_\mu^A A_\mu^A$ along the gauge orbit, namely

$$A_{\min}^2 = \min_{\{U\}} \int d^4x \text{Tr}(A_\mu^U)^2, \tag{3}$$

where U represents an element of the gauge group $SU(N)$, $A_\mu^U = U A_\mu U^{-1} + iU \partial_\mu U^{-1}$. However, for a generic choice of the gauge fixing, the operator A_{\min}^2 proves to be very difficult to be handled at quantum level. Expanding A_{\min}^2 in a power series in the gauge field, see for example [50], one obtains

$$A_{\min}^2 = \int d^4x \left[A_\mu^A \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A_\nu^A - g f^{ABC} \left(\frac{\partial_\nu}{\partial^2} \partial A^A \right) \left(\frac{1}{\partial^2} \partial A^B \right) A_\nu^C \right] + O(A^4). \tag{4}$$

The series (4) consists of an infinite number of nonlocal terms. So far, a consistent treatment for A_{\min}^2 has been achieved only in the Landau gauge, $\partial_\mu A_\mu^A = 0$, where all the nonlocal terms vanish and (4) simplifies to the already mentioned operator $\int d^4x (A_\mu^A)^2$. Let us also quote here that, in a generic linear covariant gauge, the anomalous dimension of A_{\min}^2 has been calculated at one-loop order in [51].

More recently, we have pointed out [52] that another kind of gauge invariant nonlocal operator might be relevant, namely

$$\mathcal{O} = \text{Tr} \int d^4x F_{\mu\nu} (D^2)^{-1} F_{\mu\nu}. \tag{5}$$

Unlike expression (3), operator (5) has the advantage of being localizable by means of the introduction of a suitable set of auxiliary fields. Firstly introduced in the case of 3d YM [54], the operator \mathcal{O} has received renewed interest in the context of 4d YM. In fact, we have been able to show that, when cast in local form, it gives rise to a local action which can be proven to be renormalizable to all orders in the general class of the linear covariant gauges [50]. Moreover, in [55], the anomalous dimension of (5) has been evaluated at the two-loop order in the \overline{MS} scheme and explicitly proven to be independent from the gauge parameter. Also, in the case of the Landau gauge, it has been shown [53] that the inclusion of Zwanziger's horizon function does not spoil the renormalizability of (5).

Despite the progress already achieved in the Landau and covariant linear gauges, a detailed analysis of the gauge invariant operator \mathcal{O} in the MAG is still lacking. The main task of the present paper is to fill this gap, i.e. to achieve a local and renormalizable framework for \mathcal{O} in the MAG. In particular, the manifest gauge invariance of the operator \mathcal{O} might be useful in

⁴ Here the index a runs only on the off-diagonal components, see the beginning of the following section for the notations.

order to improve our present understanding of issues such as the Abelian dominance and the dynamical gluon mass generation in the MAG.

This paper is organized as follows. In section 2, by means of the introduction of auxiliary fields, we obtain a local action for YM theories in the MAG, in the presence of the mass operator (5). Moreover, the embedding of the resulting local theory into a more general action, enables us to make use of the BRST transformations. In section 3 we obtain the full set of Ward identities fulfilled by the starting action. Section 4 is devoted to the proof of the renormalizability of this action to all orders of perturbation theory. We obtain the most general invariant counterterm and we prove that it can be reabsorbed by means of a redefinition of fields and parameters of the starting action. The last section collects our conclusions. Appendix A contains a detailed discussion of the mass operator (5) in the presence of the horizon function for the MAG.

2. Local action in the maximal Abelian gauge

As is well known, action (1) is left invariant by the gauge transformations

$$\delta_\omega A_\mu^A = -D_\mu^{AB} \omega^B, \quad (6)$$

for arbitrary $\omega^A(x)$. In order to quantize the theory, one must fix the gauge. As we shall choose an Abelian gauge, we decompose the field \mathcal{A}_μ as

$$\mathcal{A}_\mu = A_\mu^A T^A \equiv A_\mu^a T^a + A_\mu^i T^i, \quad (7)$$

with

$$[T^a, T^b] = igf^{abc} T^c + igf^{abi} T^i, \quad (8)$$

$$[T^i, T^a] = igf^{iab} T^b, \quad (9)$$

$$[T^i, T^j] = 0. \quad (10)$$

The index $i = 1, \dots, N-1$ labels the $N-1$ diagonal generators $\{T^i\}$ of the Cartan subalgebra of $SU(N)$, while the index $a = 1, \dots, N(N-1)$ labels the remaining off-diagonal generators $\{T^a\}$.

Accordingly, the field strength decomposes as

$$F_{\mu\nu}^a = D_\mu^{ab} A_\nu^b - D_\nu^{ab} A_\mu^b + gf^{abc} A_\mu^b A_\nu^c, \quad (11)$$

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + gf^{abi} A_\mu^a A_\nu^b, \quad (12)$$

where we have introduced the covariant derivative D_μ^{ab} with respect to the diagonal components A_μ^i of the gauge field, namely

$$D_\mu^{ab} \equiv \delta^{ab} \partial_\mu - gf^{abi} A_\mu^i. \quad (13)$$

2.1. The gauge fixing

The MAG [34–36] is obtained by requiring that the functional

$$\mathcal{R}[A] = \int d^4x A_\mu^a A_\mu^a \quad (14)$$

is stationary with respect to the gauge transformations. Observe that expression (14) depends only on the off-diagonal components of the gauge field. The vanishing of the first variation of \mathcal{R} leads to the nonlinear condition

$$D_\mu^{ab} A_\mu^b = 0. \quad (15)$$

Still, it remains to choose a gauge condition for the diagonal components A_μ^i of the gauge field. We shall impose a Landau-type condition, also employed in lattice numerical simulations, i.e.

$$\partial_\mu A_\mu^i = 0. \quad (16)$$

Conditions (15), (16) are implemented by adding the following gauge fixing term to the Yang–Mills action:

$$\begin{aligned} S_{\text{MAG}} = \int d^4x & (b^a D_\mu^{ab} A_\mu^b + \bar{c}^a D_\mu^{ab} D_\mu^{bc} c^c + g f^{abi} \bar{c}^a (D_\mu^{bc} A_\mu^c) c^i + \bar{c}^a D_\mu^{ab} (g f^{bcd} A_\mu^c c^d) \\ & - g^2 f^{abi} f^{cdi} \bar{c}^a c^d A_\mu^b A_\mu^c + b^i \partial_\mu A_\mu^i + \bar{c}^i \partial_\mu (\partial_\mu c^i + g f^{abi} A_\mu^a c^b)) \\ & + \alpha \int d^4x \left(\frac{1}{2} b^a b^a - g f^{abi} b^a \bar{c}^b c^i - \frac{g}{2} f^{abc} b^a \bar{c}^b c^c - \frac{g^2}{4} f^{abi} f^{cdi} \bar{c}^a \bar{c}^b c^c c^d \right. \\ & \left. - \frac{g^2}{4} f^{abc} f^{adi} \bar{c}^b \bar{c}^c c^d c^i - \frac{g^2}{8} f^{abc} f^{ade} \bar{c}^b \bar{c}^c c^d c^e \right), \end{aligned} \quad (17)$$

where $b^A \equiv (b^a, b^i)$ are the Nakanishi–Lautrup fields, and $c^A = (c^a, c^i)$, $\bar{c}^A \equiv (\bar{c}^a, \bar{c}^i)$ are the ghost and antighost fields. The gauge parameter α is introduced in (17) for renormalizability purposes. As a consequence of the nonlinearity of condition (15), the quartic interaction ghost terms in expression (17) is required in order to obtain a stable action [37, 56, 57]. After the removal of the ultraviolet divergences, the limit $\alpha \rightarrow 0$ has to be considered in order to achieve (15).

In non-Abelian gauge theories one has to face the existence of the Gribov ambiguities [3], which deeply affect the infrared region. In the MAG, it is known that condition (15) does not uniquely fix the gauge [40], so that a suitable restriction of the domain of integration in the Feynman path integral has to be implemented in order to avoid the counting of equivalent field configurations. The renormalization of operator (5) has already been investigated in the Landau gauge [53] when the restriction of the domain of integration to the so-called Gribov region was taken into account. In [53], it was explicitly shown that the introduction of the Zwanziger horizon function [4, 5], which implements the restriction to the Gribov region, does not spoil the renormalizability of (5). The same feature occurs here in the MAG. However, for simplicity, we have decided not to include in the main text the lengthy and technical analysis of the mass operator (5) in the presence of the horizon function of the MAG, leaving the inclusion of the horizon term to the appendix, where the interested reader may find a detailed discussion.

2.2. Localizing the mass operator

In [50], it has been shown that the nonlocal mass term (5) may be written in a local form by means of the introduction of a pair of complex bosonic antisymmetric tensor fields in the adjoint representation, $(B_{\mu\nu}^A, \bar{B}_{\mu\nu}^A)$, and a pair of anticommuting antisymmetric complex tensor fields, $(G_{\mu\nu}^A, \bar{G}_{\mu\nu}^A)$. Following [50], the nonlocal operator \mathcal{O} , equation (5), is coupled to the Yang–Mills action by introducing the gauge invariant mass term

$$S_{\mathcal{O}} = -\frac{m^2}{4} \int d^4x F_{\mu\nu}^A [(D^2)^{-1}]^{AB} F_{\mu\nu}^B. \quad (18)$$

Furthermore, it is easily checked that expression (18) can be rewritten in local form as

$$e^{-S_O} = \int D\bar{B}DBD\bar{G}DG \exp[-(S_{BG} + S_m)], \quad (19)$$

with

$$S_{BG} = \frac{1}{4} \int d^4x (\bar{B}_{\mu\nu}^A \mathcal{O}^{AB} B_{\mu\nu}^B - \bar{G}_{\mu\nu}^A \mathcal{O}^{AB} G_{\mu\nu}^B), \quad (20)$$

$$S_m = \frac{\text{im}}{4} \int d^4x (B_{\mu\nu}^A - \bar{B}_{\mu\nu}^A) F_{\mu\nu}^A, \quad (21)$$

and

$$\mathcal{O}^{AB} \equiv D_\mu^{AC} D_\mu^{CB}. \quad (22)$$

Identity (19) allows us to localize the expression S_O , (18), when added to the YM action S_{YM} , equation (1). Thus, for the local gauge-fixed action S_{phys} in the MAG we write

$$S_{\text{phys}} = S_{\text{YM}} + S_{\text{MAG}} + S_{BG} + S_m. \quad (23)$$

Evidently

$$\int D\bar{B}DBD\bar{G}DG \exp[-S_{\text{phys}}] = e^{-(S_{\text{YM}}+S_{\text{MAG}}+S_O)}. \quad (24)$$

2.3. A BRST invariance

In order to establish the BRST invariance of the local action S_{phys} , we shall employ here the same procedure of [5, 38, 50, 52], and we shall embed the action S_{phys} into a more general one. Following [5, 38, 50, 52], we introduce the system of external sources

$$(U_{\sigma\rho\mu\nu}, \bar{U}_{\sigma\rho\mu\nu}, V_{\sigma\rho\mu\nu}, \bar{V}_{\sigma\rho\mu\nu}) \quad (25)$$

and replace the term S_m in (23) by

$$S_{UV} = \frac{1}{4} \int d^4x F_{\mu\nu}^A (\bar{U}_{\lambda\rho\mu\nu} G_{\lambda\rho}^A + V_{\lambda\rho\mu\nu} \bar{B}_{\lambda\rho}^A - \bar{V}_{\lambda\rho\mu\nu} B_{\lambda\rho}^A + U_{\lambda\rho\mu\nu} \bar{G}_{\lambda\rho}^A). \quad (26)$$

Note that expression (21) is recovered from (26) in the physical limit for the sources, namely

$$\bar{V}_{\sigma\rho\mu\nu}|_{\text{phys}} = V_{\sigma\rho\mu\nu}|_{\text{phys}} = -\frac{\text{im}}{2} (\delta_{\sigma\mu}\delta_{\rho\nu} - \delta_{\sigma\nu}\delta_{\rho\mu}), \quad U_{\sigma\rho\mu\nu}|_{\text{phys}} = \bar{U}_{\sigma\rho\mu\nu}|_{\text{phys}} = 0, \quad (27)$$

i.e.

$$S_{UV}|_{\text{phys}} = S_m. \quad (28)$$

Thus, action (23) is replaced by

$$S_{\text{inv}} = S_{\text{YM}} + S_{\text{MAG}} + S_{BG} + S_{UV}, \quad (29)$$

which defines a more general theory and has S_{phys} as a particular case.

As an important bonus, we observe that S_{inv} is invariant under the following global $U(6)$ symmetry:

$$\mathcal{Q}_{\mu\nu\lambda\rho}(S_{\text{inv}}) = 0, \quad (30)$$

with

$$\begin{aligned} \mathcal{Q}_{\mu\nu\lambda\rho} \equiv \int d^4x \left(B_{\mu\nu}^A \frac{\delta}{\delta B_{\lambda\rho}^A} - \bar{B}_{\lambda\rho}^A \frac{\delta}{\delta \bar{B}_{\mu\nu}^A} + G_{\mu\nu}^A \frac{\delta}{\delta G_{\lambda\rho}^A} - \bar{G}_{\lambda\rho}^A \frac{\delta}{\delta \bar{G}_{\mu\nu}^A} \right. \\ \left. + U_{\mu\nu\alpha\beta} \frac{\delta}{\delta U_{\lambda\rho\alpha\beta}} - \bar{U}_{\lambda\rho\alpha\beta} \frac{\delta}{\delta \bar{U}_{\mu\nu\alpha\beta}} + V_{\mu\nu\alpha\beta} \frac{\delta}{\delta V_{\lambda\rho\alpha\beta}} - \bar{V}_{\lambda\rho\alpha\beta} \frac{\delta}{\delta \bar{V}_{\mu\nu\alpha\beta}} \right). \quad (31) \end{aligned}$$

Symmetry (30) is naturally related to the mass operator and allows us to use a multi-index notation, $(\mu, \nu) \rightarrow I, I = 1, \dots, 6$, so that

$$(B_I^A, \bar{B}_I^A, G_I^A, \bar{G}_I^A) = \frac{1}{2}(B_{\mu\nu}^A, \bar{B}_{\mu\nu}^A, G_{\mu\nu}^A, \bar{G}_{\mu\nu}^A), \quad (32)$$

$$(U_{I\mu\nu}, \bar{U}_{I\mu\nu}, V_{I\mu\nu}, \bar{V}_{I\mu\nu}) = \frac{1}{2}(U_{\sigma\rho\mu\nu}, \bar{U}_{\sigma\rho\mu\nu}, V_{\sigma\rho\mu\nu}, \bar{V}_{\sigma\rho\mu\nu}). \quad (33)$$

The use of the multi-index I will turn out to be very useful when looking for combinations of possible counterterms respecting (30). With this notation, we may rewrite action (29) as

$$S_{\text{inv}} = S_{\text{YM}} + S_{\text{MAG}} + \int d^4x (\bar{B}_I^A \mathcal{O}^{AB} B_I^B - \bar{G}_I^A \mathcal{O}^{AB} G_I^B + F_{\mu\nu}^A (\bar{U}_{I\mu\nu} G_I^A + V_{I\mu\nu} \bar{B}_I^A - \bar{V}_{I\mu\nu} B_I^A + U_{I\mu\nu} \bar{G}_I^A)). \quad (34)$$

The introduction of the system of sources (25) enables us to establish that action (29) possesses a BRST invariance. In fact, it can be checked by direct inspection that the following nilpotent transformations:

$$\begin{aligned} sA_\mu^a &= -(D_\mu^{ab} c^b + gf^{abc} A_\mu^b c^c + gf^{abi} A_\mu^b c^i), \\ sA_\mu^i &= -(\partial_\mu c^i + gf^{abi} A_\mu^a c^b), \\ sc^a &= gf^{abi} c^b c^i + \frac{g}{2} f^{abc} c^b c^c, \\ sc^i &= \frac{g}{2} f^{abi} c^a c^b, \\ s\bar{c}^A &= b^A, \quad sb^A = 0, \\ sB_I^A &= G_I^A + gf^{ABC} c^B B_I^C, \\ s\bar{B}_I^A &= gf^{ABC} c^B \bar{B}_I^C, \\ sG_I^A &= gf^{ABC} c^B G_I^C, \\ s\bar{G}_I^A &= \bar{B}_I^A + gf^{ABC} c^B \bar{G}_I^C, \\ sV_{I\mu\nu} &= U_{I\mu\nu}, \quad sU_{I\mu\nu} = 0, \\ s\bar{U}_{I\mu\nu} &= \bar{V}_{I\mu\nu}, \quad s\bar{V}_{I\mu\nu} = 0. \end{aligned} \quad (35)$$

leave action (29) invariant. In particular, we can rewrite

$$S_{\text{MAG}} = s \int d^4x \left[\bar{c}^a \left(D_\mu^{ab} + \frac{\alpha}{2} b^a \right) - \frac{\alpha}{2} gf^{abi} \bar{c}^a \bar{c}^b c^i - \frac{\alpha}{4} gf^{abc} c^a \bar{c}^b \bar{c}^c + \bar{c}^i \partial_\mu A_\mu^i \right]. \quad (36)$$

3. Identification of the complete starting action and its symmetries

In order to prove the renormalizability of the action S_{inv} , equation (29), we shall make use of the algebraic renormalization technique [58]. For that purpose, we shall introduce a set of BRST sources and we shall establish the Ward identities fulfilled by the theory. Knowledge of these Ward identities will play a central role in the determination of the most general allowed invariant counterterm.

3.1. Starting action

As is well known, due to the nonlinear character of the BRST transformations, equations (35), one has to introduce a set of external sources, $(X_I^A, \bar{X}_I^A, Y_I^A, \bar{Y}_I^A, \Omega_\mu^A, L^A)$, coupled to them,

namely

$$S_{\text{ext}} = \int d^4x \left[-\Omega_\mu^a (D_\mu^{ab} c^b + g f^{abc} A_\mu^b c^c + g f^{abi} A_\mu^b c^i) - \Omega_\mu^i (\partial_\mu c^i + g f^{abi} A_\mu^a c^b) \right. \\ \left. + L^a \left(g f^{abi} c^b c^i + \frac{g}{2} f^{abc} c^b c^c \right) + \frac{g}{2} f^{abi} L^i c^a c^b + \bar{Y}_I^A g f^{ABC} c^B B_I^C \right. \\ \left. + Y_I^A g f^{ABC} c^B \bar{B}_I^C + \bar{X}_I^A g f^{ABC} c^B G_I^C + X_I^A g f^{ABC} c^B \bar{G}_I^C \right]. \quad (37)$$

Further, we introduce the following two terms, S_λ and S_{sources} to be added to the action S_{inv} :

$$S_\lambda = \int d^4x \left\{ \lambda_1 (\bar{B}_I^A B_I^A - \bar{G}_I^A G_I^A) (\bar{V}_{J\mu\nu} V_{J\mu\nu} - \bar{U}_{J\mu\nu} U_{J\mu\nu}) \right. \\ \left. + \lambda_2 \left(\bar{B}_I^A G_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu} + \bar{G}_I^A G_J^A U_{I\mu\nu} \bar{U}_{J\mu\nu} + \bar{B}_I^A B_J^A V_{I\mu\nu} \bar{V}_{J\mu\nu} - \bar{G}_I^A B_J^A U_{I\mu\nu} \bar{V}_{J\mu\nu} \right. \right. \\ \left. - G_I^A B_J^A \bar{U}_{I\mu\nu} \bar{V}_{J\mu\nu} + \bar{G}_I^A \bar{B}_J^A U_{I\mu\nu} V_{J\mu\nu} - \frac{1}{2} B_I^A B_J^A \bar{V}_{I\mu\nu} \bar{V}_{J\mu\nu} + \frac{1}{2} G_I^A G_J^A \bar{U}_{I\mu\nu} \bar{U}_{J\mu\nu} \right. \\ \left. - \frac{1}{2} \bar{B}_I^A \bar{B}_J^A V_{I\mu\nu} V_{J\mu\nu} + \frac{1}{2} \bar{G}_I^A \bar{G}_J^A U_{I\mu\nu} U_{J\mu\nu} \right) \\ \left. + \frac{\lambda^{ABCD}}{16} (\bar{B}_I^A B_I^B - \bar{G}_I^A G_I^B) (\bar{B}_J^C B_J^D - \bar{G}_J^C G_J^D) \right\}, \quad (38)$$

$$S_{\text{sources}} = \int d^4x [\chi_1 (\bar{V}_{I\mu\nu} \partial^2 V_{I\mu\nu} - \bar{U}_{I\mu\nu} \partial^2 U_{I\mu\nu}) + \chi_2 (\bar{V}_{I\mu\nu} \partial_\mu \partial_\sigma V_{I\nu\sigma} - \bar{U}_{I\mu\nu} \partial_\mu \partial_\sigma U_{I\nu\sigma}) \\ - \zeta (\bar{U}_{I\mu\nu} U_{I\mu\nu} \bar{U}_{J\sigma\rho} U_{J\sigma\rho} + \bar{V}_{I\mu\nu} V_{I\mu\nu} \bar{V}_{J\sigma\rho} V_{J\sigma\rho} - 2 \bar{U}_{I\mu\nu} U_{I\mu\nu} \bar{V}_{J\sigma\rho} V_{J\sigma\rho})]. \quad (39)$$

The first term, S_λ , already discussed in detail in [50, 55], contains interactions between the auxiliary fields and the sources ($U_{I\mu\nu}$, $\bar{U}_{I\mu\nu}$, $V_{I\mu\nu}$, $\bar{V}_{I\mu\nu}$), and is needed for the stability of the action. The second term, S_{sources} , depends only on ($U_{I\mu\nu}$, $\bar{U}_{I\mu\nu}$, $V_{I\mu\nu}$, $\bar{V}_{I\mu\nu}$) and is allowed by power counting. The parameters λ_1 , λ_2 , χ_1 , χ_2 and ζ are free, while the 4-rank invariant tensor λ^{ABCD} enjoys the properties [50]

$$f^{MAN} \lambda^{MBCD} + f^{MBN} \lambda^{AMCD} + f^{MCN} \lambda^{ABMD} + f^{MDN} \lambda^{ABCM} = 0, \quad (40)$$

and

$$\lambda^{ABCD} = \lambda^{CDAB} = \lambda^{BACD}. \quad (41)$$

Collecting all terms, we finally write the starting action Σ as

$$\Sigma = S_{\text{inv}} + S_\lambda + S_{\text{ext}} + S_{\text{sources}}. \quad (42)$$

The stability of Σ under quantum corrections will be investigated in the following section.

Here, for the sake of clarity, we recall the range of variations of all the indices introduced so far;

$$A, B, C, D, \dots \in \{1, \dots, N^2 - 1\}, \\ a, b, c, d, \dots \in \{1, \dots, N(N - 1)\}, \\ i, j, k, l, \dots \in \{1, \dots, N - 1\}, \\ I, J, K, L, \dots \in \{1, \dots, \frac{1}{2} D(D - 1) = 6\}, \\ \mu, \nu, \sigma, \rho, \dots \in \{1, \dots, D = 4\}. \quad (43)$$

In table 1 we display the dimensions and the ghost number of the complete set of fields and sources of the theory, as well as the corresponding \mathcal{Q}_6 -charge, which is defined as the trace of operator (31).

Table 1. Quantum numbers of the fields and sources.

	A	b	\bar{c}	c	B	\bar{B}	G	\bar{G}	U	\bar{U}	V	\bar{V}	Y	\bar{Y}	X	\bar{X}	Ω	L
Dimension	1	2	2	0	1	1	1	1	1	1	1	1	3	3	3	3	3	4
Ghost number	0	0	-1	1	0	0	1	-1	1	-1	0	0	-1	-1	-2	0	-1	-2
Q_6 -charge	0	0	0	0	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	0	0

The starting action (42) is left invariant by the action of the nilpotent BRST operator s given by (35) and by

$$\begin{aligned}
 s\Omega_\mu^A &= 0, & sL^A &= 0, \\
 sY_I^A &= X_I^A, & sX_I^A &= 0, \\
 s\bar{X}_I^A &= -\bar{Y}_I^A, & s\bar{Y}_I^A &= 0,
 \end{aligned}
 \tag{44}$$

i.e.

$$s\Sigma = 0. \tag{45}$$

3.2. Ward identities

In order to constrain the possible counterterms which can be added to the local starting action Σ , equation (42), let us proceed by establishing the set of Ward identities fulfilled by the starting classical action. In fact, it turns out that Σ obeys the following set of Ward identities:

- The diagonal gauge fixing identity

$$\frac{\delta \Sigma}{\delta b^i} = \partial_\mu A_\mu^i. \tag{46}$$

- The diagonal anti-ghost equation

$$\frac{\delta \Sigma}{\delta \bar{c}^i} + \partial_\mu \frac{\delta \Sigma}{\delta \Omega_\mu^i} = 0. \tag{47}$$

- The diagonal ghost equation

$$\mathcal{G}^i(\Sigma) = \Delta_{\text{class}}^i, \tag{48}$$

where

$$\mathcal{G}^i = \frac{\delta}{\delta c^i} + gf^{abi} \bar{c}^a \frac{\delta}{\delta b^b}, \tag{49}$$

and

$$\begin{aligned}
 \Delta_{\text{class}}^i &= -\partial^2 \bar{c}^i - \partial_\mu \Omega^i + gf^{abi} \Omega_\mu^a A_\mu^b - gf^{abi} L^a c^b + gf^{abi} \bar{Y}_I^a B_I^b \\
 &\quad + gf^{abi} Y_I^a \bar{B}_I^b - gf^{abi} \bar{X}_I^a G_I^b - gf^{abi} X_I^a \bar{G}_I^b.
 \end{aligned}
 \tag{50}$$

Note that the term Δ_{class}^i , being linear in the quantum fields, represents a classical breaking not affected by quantum corrections [58].

- The Slavnov–Taylor identity

$$S(\Sigma) = 0, \tag{51}$$

where

$$\begin{aligned} \mathcal{S}(\Sigma) \equiv \int d^4x \left[\frac{\delta \Sigma}{\delta \Omega_\mu^A} \frac{\delta \Sigma}{\delta A_\mu^A} + \frac{\delta \Sigma}{\delta L^A} \frac{\delta \Sigma}{\delta c^A} + b^A \frac{\delta \Sigma}{\delta \bar{c}^A} \right. \\ + \left(\frac{\delta \Sigma}{\delta \bar{Y}_I^A} + G_I^A \right) \frac{\delta \Sigma}{\delta B_I^A} + \frac{\delta \Sigma}{\delta Y_I^A} \frac{\delta \Sigma}{\delta \bar{B}_I^A} + \frac{\delta \Sigma}{\delta \bar{X}_I^A} \frac{\delta \Sigma}{\delta G_I^A} \\ \left. + \left(\frac{\delta \Sigma}{\delta X_I^A} + \bar{B}_I^A \right) \frac{\delta \Sigma}{\delta \bar{G}_I^A} + \bar{V}_{I\mu\nu} \frac{\delta \Sigma}{\delta \bar{U}_{I\mu\nu}} + U_{I\mu\nu} \frac{\delta \Sigma}{\delta V_{I\mu\nu}} - \bar{Y}_I^A \frac{\delta \Sigma}{\delta \bar{X}_I^A} + X_I^A \frac{\delta \Sigma}{\delta Y_I^A} \right]. \end{aligned} \quad (52)$$

The Slavnov–Taylor identity (51) gives rise to the corresponding nilpotent linearized operator

$$\begin{aligned} \mathcal{S}_\Sigma \equiv \int d^4x \left[\frac{\delta \Sigma}{\delta \Omega_\mu^A} \frac{\delta}{\delta A_\mu^A} + \frac{\delta \Sigma}{\delta A_\mu^A} \frac{\delta}{\delta \Omega_\mu^A} + \frac{\delta \Sigma}{\delta L^A} \frac{\delta}{\delta c^A} + \frac{\delta \Sigma}{\delta c^A} \frac{\delta}{\delta L^A} + b^A \frac{\delta}{\delta \bar{c}^A} \right. \\ + \left(\frac{\delta \Sigma}{\delta \bar{Y}_I^A} + G_I^A \right) \frac{\delta}{\delta B_I^A} + \frac{\delta \Sigma}{\delta B_I^A} \frac{\delta}{\delta \bar{Y}_I^A} + \frac{\delta \Sigma}{\delta Y_I^A} \frac{\delta}{\delta \bar{B}_I^A} + \left(\frac{\delta \Sigma}{\delta \bar{B}_I^A} + X_I^A \right) \frac{\delta}{\delta Y_I^A} \\ + \frac{\delta \Sigma}{\delta \bar{X}_I^A} \frac{\delta}{\delta G_I^A} + \frac{\delta \Sigma}{\delta X_I^A} \frac{\delta}{\delta \bar{G}_I^A} + \left(\frac{\delta \Sigma}{\delta G_I^A} - \bar{Y}_I^A \right) \frac{\delta}{\delta \bar{X}_I^A} + \left(\frac{\delta \Sigma}{\delta X_I^A} + \bar{B}_I^A \right) \frac{\delta}{\delta \bar{G}_I^A} \\ \left. + \frac{\delta \Sigma}{\delta \bar{G}_I^A} \frac{\delta}{\delta X_I^A} + \bar{V}_{I\mu\nu} \frac{\delta}{\delta \bar{U}_{I\mu\nu}} + U_{I\mu\nu} \frac{\delta}{\delta V_{I\mu\nu}} \right], \end{aligned} \quad (53)$$

$$\mathcal{S}_\Sigma \mathcal{S}_\Sigma = 0. \quad (54)$$

- The diagonal $U(1)^{N-1}$ Ward identity

$$\mathcal{W}^i(\Sigma) = -\partial^2 b^i, \quad (55)$$

with

$$\begin{aligned} \mathcal{W}^i \equiv \partial_\mu \frac{\delta}{\delta A_\mu^i} + g f^{abi} \left(A_\mu^a \frac{\delta}{\delta A_\mu^b} + b^a \frac{\delta}{\delta b^b} + c^a \frac{\delta}{\delta c^b} + \bar{c}^a \frac{\delta}{\delta \bar{c}^b} + B_I^a \frac{\delta}{\delta B_I^b} + \bar{B}_I^a \frac{\delta}{\delta \bar{B}_I^b} + G_I^a \frac{\delta}{\delta G_I^b} \right. \\ \left. + \bar{G}_I^a \frac{\delta}{\delta \bar{G}_I^b} + \Omega_\mu^a \frac{\delta}{\delta \Omega_\mu^b} + L^a \frac{\delta}{\delta L^b} + Y_I^a \frac{\delta}{\delta Y_I^b} + \bar{Y}_I^a \frac{\delta}{\delta \bar{Y}_I^b} + X_I^a \frac{\delta}{\delta X_I^b} + \bar{X}_I^a \frac{\delta}{\delta \bar{X}_I^b} \right). \end{aligned} \quad (56)$$

- The off-diagonal $SL(2, \mathbb{R})$ identity

$$\mathcal{D}(\Sigma) = 0, \quad (57)$$

where

$$\mathcal{D}(\Sigma) = \int d^4x \left(c^a \frac{\delta \Sigma}{\delta \bar{c}^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta b^a} \right). \quad (58)$$

As the Slavnov–Taylor identity, equation (57) also defines a linear operator

$$\mathcal{D}_\Sigma \equiv \int d^4x \left(c^a \frac{\delta}{\delta \bar{c}^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta b^a} + \frac{\delta \Sigma}{\delta b^a} \frac{\delta}{\delta L^a} \right). \quad (59)$$

- The global $U(6)$ invariance related to the nonlocal mass operator

$$\mathcal{Q}_{IJ}(\Sigma) = 0, \quad (60)$$

where

$$\begin{aligned} \mathcal{Q}_{IJ} \equiv \int d^4x \left(B_I^A \frac{\delta}{\delta B_J^A} - \bar{B}_J^A \frac{\delta}{\delta \bar{B}_I^A} + G_I^A \frac{\delta}{\delta G_J^A} - \bar{G}_J^A \frac{\delta}{\delta \bar{G}_I^A} + U_{I\mu\nu} \frac{\delta}{\delta U_{J\mu\nu}} - \bar{U}_{J\mu\nu} \frac{\delta}{\delta \bar{U}_{I\mu\nu}} \right. \\ \left. + V_{I\mu\nu} \frac{\delta}{\delta V_{J\mu\nu}} - \bar{V}_{J\mu\nu} \frac{\delta}{\delta \bar{V}_{I\mu\nu}} + Y_I^A \frac{\delta}{\delta Y_J^A} - \bar{Y}_J^A \frac{\delta}{\delta \bar{Y}_I^A} + X_I^A \frac{\delta}{\delta X_J^A} - \bar{X}_J^A \frac{\delta}{\delta \bar{X}_I^A} \right). \end{aligned} \quad (61)$$

The trace of (61) defines the Q_6 -charge, $Q_6 = Q_{II}$, already displayed in table 1.

- The exact rigid symmetries

$$\mathcal{R}_{IJ}^{(\mathcal{N})}(\Sigma) = 0, \quad (62)$$

being $\mathcal{N} = 1, 2, 3, 4$ and

$$\begin{aligned} \mathcal{R}_{IJ}^{(1)} &\equiv \int d^4x \left(B_I^A \frac{\delta}{\delta G_J^A} - \bar{G}_J^A \frac{\delta}{\delta \bar{B}_I^A} + V_{I\mu\nu} \frac{\delta}{\delta U_{J\mu\nu}} - \bar{U}_{J\mu\nu} \frac{\delta}{\delta \bar{V}_{I\mu\nu}} + Y_I^A \frac{\delta}{\delta X_J^A} + \bar{X}_J^A \frac{\delta}{\delta \bar{Y}_I^A} \right), \\ \mathcal{R}_{IJ}^{(2)} &\equiv \int d^4x \left(\bar{B}_I^A \frac{\delta}{\delta \bar{G}_J^A} + G_J^A \frac{\delta}{\delta B_I^A} + \bar{V}_{I\mu\nu} \frac{\delta}{\delta \bar{U}_{J\mu\nu}} + U_{J\mu\nu} \frac{\delta}{\delta V_{I\mu\nu}} - \bar{Y}_I^A \frac{\delta}{\delta \bar{X}_J^A} + X_J^A \frac{\delta}{\delta Y_I^A} \right), \\ \mathcal{R}_{IJ}^{(3)} &\equiv \int d^4x \left(\bar{B}_I^A \frac{\delta}{\delta G_J^A} - \bar{G}_J^A \frac{\delta}{\delta B_I^A} - \bar{V}_{I\mu\nu} \frac{\delta}{\delta U_{J\mu\nu}} + \bar{U}_{J\mu\nu} \frac{\delta}{\delta V_{I\mu\nu}} + \bar{Y}_I^A \frac{\delta}{\delta X_J^A} + \bar{X}_J^A \frac{\delta}{\delta Y_I^A} \right), \\ \mathcal{R}_{IJ}^{(4)} &\equiv \int d^4x \left(B_I^A \frac{\delta}{\delta \bar{G}_J^A} + G_J^A \frac{\delta}{\delta \bar{B}_I^A} - V_{I\mu\nu} \frac{\delta}{\delta \bar{U}_{J\mu\nu}} - U_{J\mu\nu} \frac{\delta}{\delta \bar{V}_{I\mu\nu}} - Y_I^A \frac{\delta}{\delta \bar{X}_J^A} + X_J^A \frac{\delta}{\delta \bar{Y}_I^A} \right). \end{aligned} \quad (63)$$

For calculation purposes, we display the following useful (anti-)commutation relations between the linearized operator \mathcal{S}_Σ , given in (53), and operators (49), (59) and (63), namely

$$[\mathcal{D}_\Sigma, \mathcal{S}_\Sigma] = 0, \quad \{\mathcal{G}^i, \mathcal{S}_\Sigma\} = \mathcal{W}^i, \quad (64)$$

$$\{\mathcal{R}_{IJ}^{(1)}, \mathcal{S}_\Sigma\} = \mathcal{Q}_{IJ}, \quad \{\mathcal{R}_{IJ}^{(2)}, \mathcal{S}_\Sigma\} = 0, \quad (65)$$

$$\{\mathcal{R}_{IJ}^{(3)}, \mathcal{S}_\Sigma\} = \int d^4x (\delta_{IK} \delta_{JL} - \delta_{IL} \delta_{JK}) \left(\bar{B}_K^A \frac{\delta}{\delta B_L^A} - \bar{V}_{K\mu\nu} \frac{\delta}{\delta V_{L\mu\nu}} + \bar{Y}_K^A \frac{\delta}{\delta Y_L^A} \right), \quad (66)$$

$$\{\mathcal{R}_{IJ}^{(4)}, \mathcal{S}_\Sigma\} = \int d^4x (\delta_{IK} \delta_{JL} + \delta_{IL} \delta_{JK}) \left(G_K^A \frac{\delta}{\delta \bar{G}_L^A} - U_{K\mu\nu} \frac{\delta}{\delta \bar{U}_{L\mu\nu}} - X_K^A \frac{\delta}{\delta \bar{X}_L^A} \right). \quad (67)$$

4. Renormalization

In the previous section we established the full set of Ward identities fulfilled by the action Σ . In this section, we prove that Σ is perturbative renormalizable to all orders.

4.1. Determination of the most general counterterm

Let us turn our attention to the characterization of the most general invariant counterterm Σ_{CT} which can be freely added to Σ . According to the algebraic renormalization procedure [58], we require that the perturbed action $(\Sigma + \epsilon \Sigma_{CT})$ satisfy the same set of Ward identities, (46)–(62), and constraints of Σ . The counterterm Σ_{CT} must be an integrated local polynomial in the fields and sources with dimension bounded by four, vanishing ghost number and \mathcal{Q}_6 -charge, obeying the following constraints:

$$\mathcal{S}_\Sigma(\Sigma_{CT}) = 0, \quad (68)$$

$$\frac{\delta \Sigma_{CT}}{\delta b^i} = 0, \quad (69)$$

$$\frac{\delta \Sigma_{CT}}{\delta \bar{c}^i} + \partial_\mu \frac{\delta \Sigma_{CT}}{\delta \Omega_\mu^i} = 0, \quad (70)$$

$$\mathcal{D}_\Sigma(\Sigma_{CT}) = 0, \quad (71)$$

$$\mathcal{G}^i(\Sigma_{CT}) = 0, \quad (72)$$

$$\mathcal{W}^i(\Sigma_{\text{CT}}) = 0, \quad (73)$$

$$\mathcal{Q}_{IJ}(\Sigma_{\text{CT}}) = 0, \quad (74)$$

$$\mathcal{R}_{IJ}^{(N)}(\Sigma_{\text{CT}}) = 0. \quad (75)$$

As an immediate consequence of the BRST invariance, condition (68) allows us to write

$$\Sigma_{\text{CT}} = a_0 S_{\text{YM}} + \mathcal{S}_\Sigma \Delta^{(-1)}, \quad (76)$$

where a_0 is a free parameter and $\Delta^{(-1)}$ is an integrated local polynomial with ghost number -1 and vanishing \mathcal{Q}_6 -charge. Taking table 1 into account and imposing conditions (69) and (70), we are led to the expression

$$\Delta^{(-1)} = \Delta_1^{(-1)} + \Delta_2^{(-1)}, \quad (77)$$

with

$$\begin{aligned} \Delta_1^{(-1)} = & \int d^4x \{ a_1 \Omega_\mu^a A_\mu^a + a_2 (\partial_\mu \bar{c}^a) A_\mu^a + a_3 (\Omega_\mu^i + \partial_\mu \bar{c}^i) A_\mu^i + a_4 c^a L^a + a_5 c^i L^i \\ & + a_6 g f^{abi} \bar{c}^a \bar{c}^b c^i + a_7 g f^{abc} \bar{c}^a \bar{c}^b c^c + a_8 \bar{c}^a b^a + a_9 g f^{abi} \bar{c}^a A_\mu^i A_\mu^b + a_{10} \bar{Y}_I^A B_I^A \\ & + a_{11} Y_I^A \bar{B}_I^A + a_{12} \bar{X}_I^A G_I^A + a_{13} X_I^A \bar{G}_I^A + a_{14} \bar{G}_I^A \partial^2 B_I^A + a_{15} g f^{ABC} \bar{G}_I^A (\partial_\mu A_\mu^B) B_I^C \\ & + a_{16} g f^{ABC} \bar{G}_I^A A_\mu^B \partial_\mu B_I^C + a_{17} (\partial_\mu A_\nu^A) \bar{U}_{I\mu\nu} B_I^A + a_{18} A_\mu^A \bar{U}_{I\mu\nu} \partial_\nu B_I^A \\ & + a_{19} g f^{ABC} A_\mu^B A_\nu^C \bar{U}_{I\mu\nu} B_I^A + a_{20} (\partial_\mu A_\nu^A) V_{I\mu\nu} \bar{G}_I^A + a_{21} A_\mu^A V_{I\mu\nu} \partial_\nu \bar{G}_I^A \\ & + a_{22} g f^{ABC} A_\mu^B A_\nu^C V_{I\mu\nu} \bar{G}_I^A + a_{23} \bar{G}_I^A B_I^A \bar{V}_{J\mu\nu} V_{J\mu\nu} + a_{24} \bar{G}_I^A B_J^A \bar{V}_{I\mu\nu} V_{J\mu\nu} \\ & + a_{25} \bar{G}_I^A B_J^A \bar{V}_{J\mu\nu} V_{I\mu\nu} + a_{26} \bar{G}_I^A B_I^A \bar{U}_{J\mu\nu} U_{J\mu\nu} + a_{27} \bar{G}_I^A B_J^A \bar{U}_{I\mu\nu} U_{J\mu\nu} \\ & + a_{28} \bar{G}_I^A B_J^A \bar{U}_{J\mu\nu} U_{I\mu\nu} + a_{29} \bar{G}_I^A G_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu} + a_{30} \bar{G}_I^A G_J^A V_{J\mu\nu} \bar{U}_{I\mu\nu} \\ & + a_{31} \bar{B}_I^A B_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu} + a_{32} \bar{B}_I^A B_J^A V_{J\mu\nu} \bar{U}_{I\mu\nu} + a_{33} B_I^A B_I^A \bar{U}_{J\mu\nu} \bar{V}_{J\mu\nu} \\ & + a_{34} B_I^A B_J^A \bar{U}_{I\mu\nu} \bar{V}_{J\mu\nu} + a_{35} \bar{G}_I^A \bar{G}_J^A U_{I\mu\nu} V_{J\mu\nu} + a_{36} \bar{G}_I^A \bar{B}_I^A V_{J\mu\nu} V_{J\mu\nu} \\ & + a_{37} \bar{G}_I^A \bar{B}_J^A V_{I\mu\nu} V_{J\mu\nu} + a_{38} G_I^A B_J^A \bar{U}_{I\mu\nu} \bar{U}_{J\mu\nu} + \rho^{ABCD} \bar{G}_I^A B_I^B A_\mu^C A_\mu^D \\ & + a_{39} \bar{c}^a c^a \bar{U}_{I\mu\nu} V_{I\mu\nu} + \xi_1^{abcd} \bar{c}^a c^b \bar{G}_I^c B_I^d + \xi_2^{abij} \bar{c}^a c^b \bar{G}_I^i B_I^j + \sigma_1^{ABCD} \bar{G}_I^A B_I^B \bar{B}_I^C B_I^D \\ & + \sigma_2^{ABCD} \bar{G}_I^A B_I^B \bar{G}_I^C G_I^D + \sigma_3^{ABCD} \bar{G}_I^A B_J^B \bar{B}_I^C B_I^D + \sigma_4^{ABCD} \bar{G}_I^A B_J^B \bar{G}_I^C G_I^D \}, \quad (78) \end{aligned}$$

and

$$\begin{aligned} \Delta_2^{(-1)} = & \int d^4x (a_{40} \bar{U}_{I\mu\nu} \partial^2 V_{I\mu\nu} + a_{41} \bar{U}_{I\mu\nu} \partial_\mu \partial_\alpha V_{I\nu\alpha} + a_{42} \bar{U}_{I\mu\nu} V_{I\mu\nu} \bar{U}_{J\alpha\beta} U_{J\alpha\beta} \\ & + a_{43} \bar{U}_{I\mu\nu} V_{I\mu\nu} \bar{V}_{J\alpha\beta} V_{J\alpha\beta} + a_{44} \bar{U}_{I\mu\nu} V_{J\mu\nu} \bar{U}_{I\alpha\beta} U_{J\alpha\beta} + a_{45} \bar{U}_{I\mu\nu} V_{J\mu\nu} \bar{V}_{I\alpha\beta} V_{J\alpha\beta} \\ & + a_{46} \bar{U}_{I\mu\nu} V_{I\nu\alpha} \bar{U}_{J\alpha\beta} U_{J\beta\mu} + a_{47} \bar{U}_{I\mu\nu} V_{I\nu\alpha} \bar{V}_{J\alpha\beta} V_{J\beta\mu} + a_{48} \bar{U}_{I\mu\nu} V_{J\nu\alpha} \bar{U}_{I\alpha\beta} U_{J\beta\mu} \\ & + a_{49} \bar{U}_{I\mu\nu} V_{J\nu\alpha} \bar{V}_{I\alpha\beta} V_{J\beta\mu} + a_{50} V_{I\mu\nu} V_{I\mu\nu} \bar{U}_{J\alpha\beta} \bar{V}_{J\alpha\beta} + a_{51} V_{I\mu\nu} V_{J\mu\nu} \bar{U}_{I\alpha\beta} \bar{V}_{J\alpha\beta} \\ & + a_{52} \bar{U}_{I\mu\nu} \bar{U}_{J\mu\nu} V_{J\alpha\beta} U_{J\alpha\beta} + a_{53} \bar{U}_{I\mu\nu} V_{I\nu\alpha} \bar{U}_{J\mu\beta} U_{J\alpha\beta} + a_{54} \bar{U}_{I\mu\nu} V_{I\nu\alpha} \bar{V}_{J\mu\beta} V_{J\alpha\beta} \\ & + a_{55} \bar{U}_{I\mu\nu} V_{J\nu\alpha} \bar{U}_{I\mu\beta} U_{J\alpha\beta} + a_{56} \bar{U}_{I\mu\nu} V_{J\nu\alpha} \bar{V}_{I\mu\beta} V_{J\alpha\beta} + a_{57} \bar{U}_{I\mu\nu} V_{I\alpha\beta} \bar{U}_{J\alpha\beta} U_{J\mu\nu} \\ & + a_{58} \bar{U}_{I\mu\nu} V_{I\alpha\beta} \bar{V}_{J\alpha\beta} V_{J\mu\nu} + a_{59} \bar{U}_{I\alpha\mu} V_{I\nu\beta} \bar{U}_{J\alpha\beta} U_{J\mu\nu} + a_{60} \bar{U}_{I\alpha\mu} V_{I\nu\beta} \bar{V}_{J\alpha\beta} V_{J\mu\nu}) \quad (79) \end{aligned}$$

Here $a_\alpha, \alpha = 1, \dots, 60$, and the 4-rank tensor $\sigma_\aleph^{ABCD}, \aleph = 1, \dots, 4$, are also arbitrary coefficients. Applying the remaining conditions (71)–(75), expression (77) reduces to

$$\begin{aligned} \Delta^{(-1)} = & \int d^4x \left\{ a_1 \Omega_\mu^a A_\mu^a - a_2 \bar{c}^a D_\mu^{ab} A_\mu^b + a_4 c^a L^a + a_8 (\bar{c}^a b^a - g f^{abi} \bar{c}^a \bar{c}^b c^i - \frac{g}{2} f^{abc} \bar{c}^a \bar{c}^b c^c) \right. \\ & \left. + a_{10} (\bar{Y}_I^A B_I^A - \bar{X}_I^A G_I^A) + a_{11} (Y_I^A \bar{B}_I^A - X_I^A \bar{G}_I^A) + a_{14} \bar{G}_I^A D_\mu^{AC} D_\mu^{CB} B_I^B \right. \end{aligned}$$

$$\begin{aligned}
 & + a_{19} F_{\mu\nu}^A \bar{U}_{I\mu\nu} B_I^A + (a_{10} - a_{11} - a_{19}) F_{\mu\nu}^A V_{I\mu\nu} \bar{G}_I^A + a_{23} \bar{G}_I^A B_I^A (\bar{V}_{J\mu\nu} V_{J\mu\nu} - \bar{U}_{J\mu\nu} U_{J\mu\nu}) \\
 & + a_{25} (\bar{G}_I^A B_J^A \bar{V}_{J\mu\nu} V_{I\mu\nu} + \bar{G}_I^A G_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu}) + a_{28} (\bar{G}_I^A B_J^A \bar{U}_{J\mu\nu} U_{I\mu\nu} + \bar{B}_I^A B_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu}) \\
 & + \frac{1}{2} [\lambda_2 (a_{10} - a_{11}) - a_{28} - a_{25}] (B_I^A B_J^A \bar{U}_{I\mu\nu} \bar{V}_{J\mu\nu} + G_I^A B_J^A \bar{U}_{I\mu\nu} \bar{U}_{J\mu\nu}) \\
 & - \frac{1}{2} [\lambda_2 (a_{10} - a_{11}) + a_{28} + a_{25}] (\bar{G}_I^A \bar{G}_J^A U_{I\mu\nu} V_{J\mu\nu} + \bar{G}_I^A \bar{B}_J^A V_{I\mu\nu} V_{J\mu\nu}) \\
 & + \sigma_1^{ABCD} \bar{G}_I^A B_I^B (\bar{B}_J^C B_J^D - \bar{G}_J^C G_J^D) + a_{40} \bar{U}_{I\mu\nu} \partial^2 V_{I\mu\nu} + a_{41} \bar{U}_{I\mu\nu} \partial_\mu \partial_\alpha V_{I\nu\alpha} \\
 & + a_{42} (\bar{U}_{I\mu\nu} V_{I\mu\nu} \bar{U}_{J\alpha\beta} U_{J\alpha\beta} - \bar{U}_{I\mu\nu} V_{I\mu\nu} \bar{V}_{J\alpha\beta} V_{J\alpha\beta}) \Big\}, \tag{80}
 \end{aligned}$$

with the further restrictions on σ_1^{ABCD}

$$f^{MAN} \sigma_1^{MBCD} + f^{MBN} \sigma_1^{AMCD} + f^{MCN} \sigma_1^{ABMD} + f^{MDN} \sigma_1^{ABCM} = 0, \tag{81}$$

and

$$\sigma_1^{ABCD} = \sigma_1^{CDAB} = \sigma_1^{BACD}. \tag{82}$$

Observe that many coefficients present in expressions (78) and (79) vanish after the imposition of constraints (71)–(75).

Performing now the following redefinition:

$$\begin{aligned}
 a_0 & \rightarrow a_0, & a_1 & \rightarrow a_1, \\
 a_2 & \rightarrow -a_2, & a_4 & \rightarrow -a_3, \\
 a_8 & \rightarrow \frac{\alpha}{2} a_4, & a_{10} + a_{11} & \rightarrow a_5, \\
 a_{10} - a_{19} & \rightarrow a_6, & a_{23} & \rightarrow \lambda_1 a_7, \\
 a_{25} + a_{28} & \rightarrow \lambda_2 a_8, & a_{40} & \rightarrow \chi_1 a_9, \\
 a_{41} & \rightarrow \chi_2 a_{10}, & a_{42} & \rightarrow \zeta a_{11}, \\
 \sigma_1^{ABCD} & \rightarrow \frac{a_5}{16} (\sigma^{ABCD} - \lambda^{ABCD}),
 \end{aligned} \tag{83}$$

we can rewrite the counterterm Σ_{CT} as

$$\Sigma_{CT} = \Sigma_{CT(1)} + \Sigma_{CT(2)} + \Sigma_{CT(3)} \tag{84}$$

with

$$\begin{aligned}
 \Sigma_{CT(1)} & = \int d^4x \Big\{ \frac{a_0 + 2a_1}{2} (D_\mu^{ab} A_\nu^b) (D_\mu^{ac} A_\nu^c - D_\nu^{ac} A_\mu^c) + (a_0 + 3a_1) g f^{abc} (D_\mu^{ad} A_\nu^d) A_\mu^b A_\nu^c \\
 & + \frac{a_0 + 4a_1}{4} (g^2 f^{abc} f^{ade} A_\mu^b A_\nu^c A_\mu^d A_\nu^e + g^2 f^{abi} f^{cdi} A_\mu^a A_\nu^b A_\mu^c A_\nu^d) \\
 & + \frac{a_0}{2} (\partial_\mu A_\nu^i) (\partial_\mu A_\nu^i - \partial_\nu A_\mu^i) + (a_0 + 2a_1) g f^{abi} (\partial_\mu A_\nu^i) A_\mu^a A_\nu^b + (a_1 + a_2) b^a D_\mu^{ab} A_\mu^a \\
 & + (a_1 + a_2) g f^{abi} \bar{c}^a (D_\mu^{bc} A_\mu^c) c^i + (a_1 + a_2 + a_3) \bar{c}^a D_\mu^{ab} (g f^{bcd} A_\mu^c c^d) \\
 & + (2a_1 + a_2 + a_3) g^2 f^{abi} f^{cdi} \bar{c}^a c^d A_\mu^b A_\mu^c - (a_1 + a_3) (\Omega_\mu^i + \partial_\mu \bar{c}^i) g f^{abi} A_\mu^a c^b \\
 & + (a_2 + a_3) \bar{c}^a D_\mu^{ab} D_\mu^{bc} c^c + a_4 \frac{\alpha}{2} b^a b^a - a_4 \alpha g f^{abi} b^a \bar{c}^b c^i \\
 & - (a_3 + a_4) \left[\frac{\alpha}{2} g f^{abc} b^a \bar{c}^b c^c + \frac{\alpha}{4} g^2 f^{abc} f^{adi} \bar{c}^b \bar{c}^c c^d c^i \right] \\
 & - (2a_3 + a_4) \left[\frac{\alpha}{4} g^2 f^{abi} f^{cdi} \bar{c}^a \bar{c}^b c^c c^d + \frac{\alpha}{8} g^2 f^{abc} f^{ade} \bar{c}^b \bar{c}^c c^d c^e \right] + (a_1 - a_3) \Omega_\mu^a D_\mu^{ab} c^b \\
 & - a_3 g f^{abc} \Omega_\mu^a A_\mu^b c^c + a_3 \frac{g}{2} f^{abc} L^a c^b c^c + a_3 g f^{abi} L^i c^a c^b \Big\}, \tag{85}
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{\text{CT}(2)} = \int d^4x \left\{ a_3 \bar{Y}_I^a (g f^{abc} c^b B_I^c + g f^{abi} c^b B_I^i) + a_3 f^{abi} \bar{Y}_I^i c^a B_I^b + a_3 Y_I^a (g f^{abc} c^b \bar{B}_I^c \right. \\
 + g f^{abi} c^b \bar{B}_I^i) + a_3 g f^{abi} Y_I^i c^a \bar{B}_I^b + a_3 \bar{X}_I^a (g f^{abc} c^b G_I^c + g f^{abi} c^b G_I^i) \\
 + a_3 g f^{abi} \bar{X}_I^i c^a G_I^b + a_3 X_I^a (g f^{abc} c^b \bar{G}_I^c + g f^{abi} c^b \bar{G}_I^i) + a_3 g f^{abi} X_I^i c^a \bar{G}_I^b \\
 + a_5 \bar{B}_I^a \partial^2 B_I^a - a_5 g f^{abi} \bar{B}_I^a (\partial_\mu A_\mu^i) B_I^b - (a_1 + a_5) g f^{abc} \bar{B}_I^a (\partial_\mu A_\mu^c) B_I^b \\
 - a_5 2g f^{abi} \bar{B}_I^a A_\mu^i \partial_\mu B_I^b - (a_1 + a_5) 2g f^{abc} \bar{B}_I^a A_\mu^c \partial_\mu B_I^b + a_5 g^2 f^{aci} f^{cbj} \bar{B}_I^a A_\mu^i A_\mu^j B_I^b \\
 + (a_1 + a_5) g^2 f^{aci} f^{cbd} \bar{B}_I^a A_\mu^i A_\mu^d B_I^b + (a_1 + a_5) g^2 f^{acd} f^{cdi} \bar{B}_I^a A_\mu^d A_\mu^i B_I^b \\
 + (2a_1 + a_5) g^2 f^{acd} f^{cbe} \bar{B}_I^a A_\mu^d A_\mu^e B_I^b + (2a_1 + a_5) g^2 f^{aci} f^{bdi} \bar{B}_I^a A_\mu^c A_\mu^d B_I^b \\
 + a_5 \bar{B}_I^i \partial^2 B_I^i - (2a_1 + a_5) g^2 f^{abi} f^{acj} \bar{B}_I^i A_\mu^b A_\mu^c B_I^j + (a_1 + a_5) g f^{abi} \bar{B}_I^i (\partial_\mu A_\mu^b) B_I^i \\
 + (a_1 + a_5) 2g f^{abi} \bar{B}_I^i A_\mu^b \partial_\mu B_I^i - (a_1 + a_5) g^2 f^{acj} f^{cbi} \bar{B}_I^i A_\mu^j A_\mu^b B_I^i \\
 - (2a_1 + a_5) g^2 f^{acb} f^{cdi} \bar{B}_I^i A_\mu^c A_\mu^d B_I^i - (a_1 + a_5) g f^{abi} \bar{B}_I^i (\partial_\mu A_\mu^b) B_I^a \\
 - (a_1 + a_5) 2g f^{abi} \bar{B}_I^i A_\mu^b \partial_\mu B_I^a + (a_1 + a_5) g^2 f^{cbi} f^{caj} \bar{B}_I^i A_\mu^c A_\mu^j B_I^a \\
 \left. + (2a_1 + a_5) g^2 f^{cbi} f^{cad} \bar{B}_I^i A_\mu^c A_\mu^d B_I^a \right\}, \tag{86}
 \end{aligned}$$

and

$$\begin{aligned}
 \Sigma_{\text{CT}(3)} = \int d^4x \left\{ -[a_5 \bar{G}_I^a \partial^2 G_I^a - a_5 g f^{abi} \bar{G}_I^a (\partial_\mu A_\mu^i) G_I^b - (a_1 + a_5) g f^{abc} \bar{G}_I^a (\partial_\mu A_\mu^c) G_I^b \right. \\
 - a_5 2g f^{abi} \bar{G}_I^a A_\mu^i \partial_\mu G_I^b - (a_1 + a_5) 2g f^{abc} \bar{G}_I^a A_\mu^c \partial_\mu G_I^b + a_5 g^2 f^{aci} f^{cbj} \bar{G}_I^a A_\mu^i A_\mu^j G_I^b \\
 + (a_1 + a_5) g^2 f^{aci} f^{cbd} \bar{G}_I^a A_\mu^i A_\mu^d G_I^b + (a_1 + a_5) g^2 f^{acd} f^{cdi} \bar{G}_I^a A_\mu^d A_\mu^i G_I^b \\
 + (2a_1 + a_5) g^2 f^{acd} f^{cbe} \bar{G}_I^a A_\mu^d A_\mu^e G_I^b + (2a_1 + a_5) g^2 f^{aci} f^{bdi} \bar{G}_I^a A_\mu^c A_\mu^d G_I^b \\
 + a_5 \bar{G}_I^i \partial^2 G_I^i - (2a_1 + a_5) g^2 f^{abi} f^{acj} \bar{G}_I^i A_\mu^b A_\mu^c G_I^j + (a_1 + a_5) g f^{abi} \bar{G}_I^i (\partial_\mu A_\mu^b) G_I^i \\
 + (a_1 + a_5) 2g f^{abi} \bar{G}_I^i A_\mu^b \partial_\mu G_I^i - (a_1 + a_5) g^2 f^{acj} f^{cbi} \bar{G}_I^i A_\mu^j A_\mu^b G_I^i \\
 - (2a_1 + a_5) g^2 f^{acb} f^{cdi} \bar{G}_I^i A_\mu^c A_\mu^d G_I^i - (a_1 + a_5) g f^{abi} \bar{G}_I^i (\partial_\mu A_\mu^b) G_I^a \\
 - (a_1 + a_5) 2g f^{abi} \bar{G}_I^i A_\mu^b \partial_\mu G_I^a + (a_1 + a_5) g^2 f^{cbi} f^{caj} \bar{G}_I^i A_\mu^c A_\mu^j G_I^a \\
 + (2a_1 + a_5) g^2 f^{cbi} f^{cad} \bar{G}_I^i A_\mu^c A_\mu^d G_I^a] + [2(a_1 + a_6) D_\mu^{ab} A_\nu^b + (2a_1 + a_6) g f^{abc} A_\mu^a A_\nu^c] \\
 \times (\bar{U}_{I\mu\nu} G_I^a + V_{I\mu\nu} \bar{B}_I^a - \bar{V}_{I\mu\nu} B_I^a + U_{I\mu\nu} \bar{G}_I^a) + [2a_6 \partial_\mu A_\nu^i + (2a_1 + a_6) g f^{abi} A_\mu^a A_\nu^b] \\
 \times (\bar{U}_{I\mu\nu} G_I^i + V_{I\mu\nu} \bar{B}_I^i - \bar{V}_{I\mu\nu} B_I^i + U_{I\mu\nu} \bar{G}_I^i) + \lambda_1 (a_5 + a_7) (\bar{B}_I^A B_I^A - \bar{G}_I^A G_I^A) \\
 \times (\bar{V}_{J\mu\nu} V_{J\mu\nu} - \bar{U}_{J\mu\nu} U_{J\mu\nu}) + \lambda_2 (a_5 + a_8) \left(\bar{B}_I^A B_J^A V_{I\mu\nu} \bar{V}_{J\mu\nu} - \bar{G}_I^A B_J^A U_{I\mu\nu} \bar{V}_{J\mu\nu} \right. \\
 - G_I^A B_J^A \bar{U}_{I\mu\nu} \bar{V}_{J\mu\nu} + \bar{B}_I^A G_J^A V_{I\mu\nu} \bar{U}_{J\mu\nu} + \bar{G}_I^A G_J^A U_{I\mu\nu} \bar{U}_{J\mu\nu} + \bar{G}_I^A \bar{B}_J^A U_{I\mu\nu} V_{J\mu\nu} \\
 \left. - \frac{1}{2} \bar{B}_I^A B_J^A \bar{V}_{I\mu\nu} \bar{V}_{J\mu\nu} + \frac{1}{2} G_I^A G_J^A \bar{U}_{I\mu\nu} \bar{U}_{J\mu\nu} - \frac{1}{2} \bar{B}_I^A \bar{B}_J^A V_{I\mu\nu} V_{J\mu\nu} + \frac{1}{2} \bar{G}_I^A \bar{G}_J^A U_{I\mu\nu} U_{J\mu\nu} \right) \\
 + \frac{a_5}{16} (\lambda^{ABCD} + \sigma^{ABCD}) (\bar{B}_I^A B_I^B - \bar{G}_I^A G_I^B) (\bar{B}_J^C B_J^D - \bar{G}_J^C G_J^D) \\
 + \chi_1 a_9 (\bar{V}_{I\mu\nu} \partial^2 V_{I\mu\nu} - \bar{U}_{I\mu\nu} \partial^2 U_{I\mu\nu}) + \chi_2 a_{10} (\bar{V}_{I\mu\nu} \partial_\mu \partial_\alpha V_{I\nu\alpha} - \bar{U}_{I\mu\nu} \partial_\mu \partial_\alpha U_{I\nu\alpha}) \\
 \left. - \zeta a_{11} (\bar{U}_{I\mu\nu} U_{I\mu\nu} \bar{U}_{J\alpha\beta} U_{J\alpha\beta} + \bar{V}_{I\mu\nu} V_{I\mu\nu} \bar{V}_{J\alpha\beta} V_{J\alpha\beta} - 2 \bar{V}_{I\mu\nu} V_{I\mu\nu} \bar{U}_{J\alpha\beta} U_{J\alpha\beta}) \right\}. \tag{87}
 \end{aligned}$$

By construction, expression (84) yields the most general invariant counterterm compatible with the full set of Ward identities.

4.2. Renormalization factors

Once we have found the most general counterterm, equation (84), we have to check whether the remaining independent coefficients, a_0, a_1, \dots, a_{11} , and the 4-rank tensor σ^{ABCD} can be reabsorbed through a redefinition of the fields, sources and parameters of the starting action Σ . The answer is in fact affirmative. Let us rename collectively the fields, sources and parameters as

$$\Phi = (A, b, \bar{c}, c) \quad \text{and} \quad \Psi = (\bar{B}, B, \bar{G}, G), \quad (88)$$

$$J = (\Omega, L) \quad \text{and} \quad \mathcal{J} = (\bar{Y}, Y, \bar{X}, X, \bar{U}, U, \bar{V}, V), \quad (89)$$

$$\xi = (g, \alpha, \lambda_1, \lambda_2, \chi_1, \chi_2, \zeta). \quad (90)$$

Next, defining the bare fields, sources and parameters as

$$\begin{aligned} \Phi_0^{\text{off-diag}} &= \tilde{Z}_\Phi^{1/2} \Phi^{\text{off-diag}}, & \Phi_0^{\text{diag}} &= Z_\Phi^{1/2} \Phi^{\text{diag}}, \\ \Psi_0 &= Z_\Psi^{1/2} \Psi, & J_0^{\text{off-diag}} &= \tilde{Z}_J J^{\text{off-diag}}, \\ J_0^{\text{diag}} &= Z_J J^{\text{diag}}, & \mathcal{J}_0 &= Z_\mathcal{J} \mathcal{J}, \\ \xi_0 &= Z_\xi \xi, \end{aligned} \quad (91)$$

and

$$\lambda_0^{ABCD} = Z_\lambda \lambda^{ABCD} + \mathcal{Z}^{ABCD}, \quad (92)$$

it is easily checked that the invariant counterterm Σ_{CT} can be reabsorbed into the starting classical action Σ , namely

$$\Sigma[\xi, \Phi, \Psi, J, \mathcal{J}, \lambda^{ABCD}] + \epsilon \Sigma_{\text{CT}} = \Sigma[\xi_0, \Phi_0, \Psi_0, J_0, \mathcal{J}_0, \lambda_0^{ABCD}] + O(\epsilon^2), \quad (93)$$

where ϵ stands for an infinitesimal expansion parameter. For the Z 's factors we have

$$\begin{aligned} Z_A^{1/2} &= Z_g^{-1}, & \tilde{Z}_b^{1/2} &= Z_g Z_c^{1/2} \tilde{Z}_c^{1/2}, \\ Z_b^{1/2} &= Z_g, & \tilde{Z}_{\bar{c}}^{1/2} &= \tilde{Z}_c^{1/2}, \\ Z_{\bar{c}}^{1/2} &= Z_{\bar{c}}^{-1/2}, & Z_{\bar{B}}^{1/2} &= Z_{\bar{G}}^{1/2} = Z_G^{1/2} = Z_B^{1/2}, \\ \tilde{Z}_\Omega &= Z_g^{-1} Z_c^{-1/2} \tilde{Z}_A^{-1/2}, & Z_\Omega &= Z_c^{-1/2}, \\ \tilde{Z}_L &= Z_g^{-1} Z_c^{-1/2} \tilde{Z}_c^{-1/2}, & Z_L &= Z_g^{-1} Z_c^{-1}, \\ Z_X &= Z_{\bar{X}} = Z_Y = Z_{\bar{Y}} = Z_g^{-1} Z_c^{-1/2} Z_B^{-1/2}, & Z_U &= Z_{\bar{U}} = Z_{\bar{V}} = Z_V, \end{aligned} \quad (94)$$

with

$$\begin{aligned} \tilde{Z}_A^{1/2} &= 1 + \epsilon \left(\frac{a_0}{2} + a_1 \right), & Z_g &= 1 - \epsilon \frac{a_0}{2}, \\ \tilde{Z}_c^{1/2} &= 1 + \epsilon \frac{a_2 + a_3}{2}, & Z_c^{1/2} &= 1 + \epsilon \frac{a_2 - a_3}{2}, \\ Z_B^{1/2} &= 1 + \epsilon \frac{a_5}{2}, & Z_V &= 1 - \epsilon \left(\frac{a_0}{2} + \frac{a_5}{2} - a_6 \right), \\ Z_\alpha &= 1 + \epsilon (a_0 - 2a_2 + a_4), & Z_{\lambda_1} &= 1 + \epsilon (a_0 + a_5 - 2a_6 + a_7), \\ Z_{\lambda_2} &= 1 + \epsilon (a_0 + a_5 - 2a_6 + a_8), & Z_{\chi_1} &= 1 + \epsilon (a_0 + a_5 - 2a_6 + a_9), \\ Z_{\chi_2} &= 1 + \epsilon (a_0 + a_5 - 2a_6 + a_{10}), & Z_\zeta &= 1 + \epsilon (2a_0 + 2a_5 - 4a_6 + a_{11}), \\ Z_\lambda &= 1 - \epsilon a_5, & \mathcal{Z}^{ABCD} &= \epsilon a_5 \sigma^{ABCD}. \end{aligned} \quad (95)$$

This concludes the proof of the renormalizability of the classical action to all orders of perturbation theory.

5. Conclusions

In this paper, a detailed analysis of the nonlocal gauge invariant mass operator $\text{Tr} \int d^4x F_{\mu\nu} (D^2)^{-1} F_{\mu\nu}$ has been made in the MAG. By means of the introduction of a suitable set of auxiliary fields, this operator can be cast in local form. Moreover, the embedding of the resulting local model into a more general action has allowed us to make use of the BRST symmetry. Furthermore, it turns out that the generalized action displays additional global symmetries giving rise to useful Ward identities, which were used to restrict the possible counterterms. The analysis of the renormalization factors has enabled us to show that the most general invariant counterterm can be in fact reabsorbed into the starting action through a redefinition of fields, parameter and sources, establishing thus the perturbative renormalizability of the model to all orders. Finally, in the appendix the nonlocal operator $\text{Tr} \int d^4x F_{\mu\nu} (D^2)^{-1} F_{\mu\nu}$ has been analyzed in the presence of the horizon function implementing the restriction of the domain of integration in the Feynman path integral to the Gribov region in the MAG. The output of our analysis is that the introduction of the horizon function does not spoil the renormalizability of the model.

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Appendix A. Including the horizon function

It is a well-known fact that non-Abelian theories are plagued by Gribov ambiguities [3], see [60] for a pedagogical review. In the specific case of the MAG, the study of Gribov ambiguities and the characterization of the horizon function in the case of $SU(2)$ can be found in [39, 38]. In this appendix, we present the main aspects of the simultaneous inclusion of the MAG horizon function and of the gauge invariant mass operator (5) in the YM theory. A similar treatment regarding Gribov ambiguities and the mass operator (5) has been done recently in the Landau gauge [53]. Without loss of generality, we will follow [39, 38] and restrict ourselves to $SU(2)$.

In [38], the following horizon function for the MAG has been derived

$$S_{\text{Horizon}} = \gamma^4 g^2 \int d^4x \varepsilon^{ab} A_\mu (\mathcal{M}^{-1})^{ac} \varepsilon^{cb} A_\mu. \quad (\text{A.1})$$

Here γ stands for the Gribov parameter [3, 39], $\varepsilon^{ab} \equiv \varepsilon^{3ab}$, $a, b = 1, 2$ are the off-diagonal components of the $SU(2)$ structure constants and $A_\mu \equiv A_\mu^3$ is the diagonal component of the gauge field. The operator $(\mathcal{M}^{-1})^{ab}$ is the inverse of the Faddeev–Popov operator given by

$$\mathcal{M}^{ab} = -D_\mu^{ac} D_\mu^{cb} - g^2 \varepsilon^{ac} \varepsilon^{bd} A_\mu^c A_\mu^d, \quad (\text{A.2})$$

with the covariant derivative D_μ^{ab} defined as a particular case of (13) by

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - g \varepsilon^{ab} A_\mu. \quad (\text{A.3})$$

The inclusion of the horizon function (A.1) allows one to implement the restriction of the domain of integration in the Feynman path integral to the Gribov region, where operator (A.2) is strictly positive definite. As underlined in [38, 39], such a restriction is necessary in order to deal with the Gribov copies.

In much the same way as the gauge invariant mass operator (5), the horizon function (A.1) also possesses a localized version, which can be obtained through the introduction of a suitable pair of commuting auxiliary complex fields $(\phi_\mu^{ab}, \bar{\phi}_\mu^{ab})$, and a pair of anti-commuting ones $(\omega_\mu^{ab}, \bar{\omega}_\mu^{ab})$ [38]

$$\begin{aligned}
 S_{\text{Horizon}}^{\text{local}} = & \int d^4x \left\{ \bar{\phi}_\mu^{ab} \mathcal{M}^{ac} \phi_\mu^{cb} - \bar{\omega}_\mu^{ab} \mathcal{M}^{ac} \omega_\mu^{cb} + \bar{\omega}_\mu^{ab} \mathcal{F}^{ac} \phi_\mu^{cb} + \bar{M}_{\mu\nu}^{ac} D_\mu^{ab} \phi_\nu^{bc} + N_{\mu\nu}^{ac} D_\mu^{ab} \bar{\omega}_\nu^{bc} \right. \\
 & + \bar{N}_{\mu\nu}^{ac} \left[D_\mu^{ab} \omega_\nu^{bc} + g \varepsilon^{ab} (\partial_\mu c + g \varepsilon^{de} A_\mu^d c^e) \phi_\nu^{bc} \right] \\
 & + M_{\mu\nu}^{ac} \left[D_\mu^{ab} \bar{\phi}_\nu^{bc} + g \varepsilon^{ab} (\partial_\mu c + g \varepsilon^{de} A_\mu^d c^e) \bar{\omega}_\nu^{bc} \right] \left. \right\} \\
 & + \alpha g^2 \int d^4x \left[\frac{1}{2} (\bar{\phi}_\mu^{ac} \phi_\mu^{ac} - \bar{\omega}_\mu^{ac} \omega_\mu^{ac}) (\bar{\phi}_\nu^{bd} \phi_\nu^{bd} - \bar{\omega}_\nu^{bd} \omega_\nu^{bd}) - (\bar{\phi}_\mu^{ac} \phi_\mu^{ac} - \bar{\omega}_\mu^{ac} \omega_\mu^{ac}) \bar{c}^b c^b \right. \\
 & \left. + \bar{\omega}_\mu^{ac} \phi_\mu^{ac} b^b c^b - \bar{\omega}_\mu^{ac} \phi_\mu^{ac} g \varepsilon^{bd} \bar{c}^b c^d c \right], \tag{A.4}
 \end{aligned}$$

with \mathcal{F}^{ab} given by

$$\begin{aligned}
 \mathcal{F}^{ab} = & 2g \varepsilon^{ac} (\partial_\mu c + g \varepsilon^{de} A_\mu^d c^e) D_\mu^{cb} + g \varepsilon^{ab} \partial_\mu (\partial_\mu c + g \varepsilon^{cd} A_\mu^c c^d) \\
 & - g^2 (\varepsilon^{ac} \varepsilon^{bd} + \varepsilon^{ad} \varepsilon^{bc}) A_\mu^d (D_\mu^{ce} c^e + g \varepsilon^{ce} A_\mu^e c), \tag{A.5}
 \end{aligned}$$

and the sources $M_{\mu\nu}^{ab}$, $\bar{M}_{\mu\nu}^{ab}$, $N_{\mu\nu}^{ab}$ and $\bar{N}_{\mu\nu}^{ab}$ are chosen in such a way that their physical values must be taken as

$$\bar{M}_{\mu\nu}^{ab}|_{\text{phys}} = -M_{\mu\nu}^{ab}|_{\text{phys}} = \delta^{ab} \delta_{\mu\nu} \gamma^2, \quad \bar{N}_{\mu\nu}^{ab}|_{\text{phys}} = N_{\mu\nu}^{ab}|_{\text{phys}} = 0. \tag{A.6}$$

Note that the last term in expression (A.4) introduces quartic interactions between the auxiliary fields, being needed for renormalizability. Nevertheless, unlike the term (38), these quartic terms depend on the gauge parameter α of (17), which will be set to zero, $\alpha \rightarrow 0$, after the removal of the ultraviolet divergences.

Adding the local version of the horizon function (A.4) to action (42), we obtain a new starting action which reads

$$S = S_{\text{inv}} + S_{\text{horizon}}^{\text{local}} + S_\lambda + \tilde{S}_{\text{sources}} + \tilde{S}_{\text{ext}}. \tag{A.7}$$

The first three terms of expression (A.7) are given by (29), (A.4) and (38) respectively, while the fourth term generalizes (39), including the new sources (A.6), namely

$$\tilde{S}_{\text{sources}} = S_{\text{sources}} + \int d^4x \chi (\bar{M}_{\mu\nu}^{ab} M_{\mu\nu}^{ab} + \bar{N}_{\mu\nu}^{ab} N_{\mu\nu}^{ab}). \tag{A.8}$$

Finally, the last term of (A.7) contains the coupling of the external sources Ω_μ^a , τ_μ^a , ξ_μ^a , Ω_μ , L^a , L , \bar{Y}_I^B , Y_I^B , \bar{X}_I^B , X_I^B and λ_μ^{ac} , η_μ^{ae} , ρ_μ^{ac} , ϑ_μ^{ae} to some nonlinear operators needed for the BRST invariance of the model, being given by

$$\begin{aligned}
 \tilde{S}_{\text{ext}} = & \int d^4x \left\{ -\Omega_\mu^a D_\mu^{ab} c^b - g \varepsilon^{ab} \tau_\mu^a A_\mu^b c + \xi_\mu^a \left[g \varepsilon^{ab} (D_\mu^{bc} c^c) c - \frac{g^2}{2} \varepsilon^{ab} \varepsilon^{cd} A_\mu^b c^c c^d \right] \right. \\
 & - \Omega_\mu (\partial_\mu c + g \varepsilon^{ab} A_\mu^a c^b) + g \varepsilon^{ab} L^a c^b c + \frac{g}{2} \varepsilon^{ab} L c^a c^b \\
 & + g \varepsilon^{ABC} c^A (\bar{Y}_I^B B_I^C + Y_I^B \bar{B}_I^C - \bar{X}_I^B G_I^C - X_I^B \bar{G}_I^C) + g \varepsilon^{ab} \lambda_\mu^{ac} \phi_\mu^{bc} c \\
 & + \eta_\mu^{ae} \left[g \varepsilon^{ab} \omega_\mu^{be} c + \frac{g^2}{2} \varepsilon^{ab} \varepsilon^{cd} \phi_\mu^{be} c^c c^d \right] + g \varepsilon^{ab} \rho_\mu^{ac} \bar{\omega}_\mu^{bc} c \\
 & \left. - \vartheta_\mu^{ae} \left[g \varepsilon^{ab} \bar{\phi}_\mu^{be} c - \frac{g^2}{2} \varepsilon^{ab} \varepsilon^{cd} \bar{\omega}_\mu^{be} c^c c^d \right] \right\}. \tag{A.9}
 \end{aligned}$$

Table A1. Quantum numbers of the fields and sources.

	ϕ	$\bar{\phi}$	ω	$\bar{\omega}$	M	\bar{M}	N	\bar{N}	ϑ	η	ρ	λ	Ω	τ	ξ
Dimension	1	1	1	1	2	2	2	2	3	3	3	3	3	3	3
Ghost number	0	0	1	-1	0	0	1	-1	-1	-2	0	-1	-1	-1	-2
Q_8 -charge	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	0	0	0

The quantum numbers of the new fields and sources are displayed in table A1.

Let us now proceed by giving the set of Ward identities fulfilled by action (A.7). These are

- The Slavnov–Taylor identity

$$\begin{aligned}
 S(S) = \int d^4x \left[\left(\frac{\delta S}{\delta \Omega_\mu^a} + \frac{\delta S}{\delta \tau_\mu^a} \right) \frac{\delta S}{\delta A_\mu^a} + \frac{\delta S}{\delta \Omega_\mu} \frac{\delta S}{\delta A_\mu} + \frac{\delta S}{\delta L^A} \frac{\delta S}{\delta c^A} + b^A \frac{\delta S}{\delta \bar{c}^A} + \left(\frac{\delta S}{\delta \bar{Y}_I^A} + G_I^A \right) \frac{\delta S}{\delta B_I^A} \right. \\
 + \frac{\delta S}{\delta Y_I^A} \frac{\delta S}{\delta \bar{B}_I^A} + \frac{\delta S}{\delta \bar{X}_I^A} \frac{\delta S}{\delta G_I^A} + \left(\frac{\delta S}{\delta X_I^A} + \bar{B}_I^A \right) \frac{\delta S}{\delta \bar{G}_I^A} + \bar{V}_{I\mu\nu} \frac{\delta S}{\delta \bar{U}_{I\mu\nu}} + U_{I\mu\nu} \frac{\delta S}{\delta V_{I\mu\nu}} \\
 - \bar{Y}_I^A \frac{\delta S}{\delta \bar{X}_I^A} + X_I^A \frac{\delta S}{\delta Y_I^A} + \omega_\mu^{ab} \frac{\delta S}{\delta \phi_\mu^{ab}} + \bar{\phi}_\mu^{ab} \frac{\delta S}{\delta \bar{\omega}_\mu^{ab}} + N_{\mu\nu}^{ab} \frac{\delta S}{\delta M_{\mu\nu}^{ab}} \\
 \left. - \bar{M}_{\mu\nu}^{ab} \frac{\delta S}{\delta \bar{N}_{\mu\nu}^{ab}} - (\Omega_\mu^a - \tau_\mu^a) \frac{\delta S}{\delta \xi_\mu^a} + \lambda_\mu^{ab} \frac{\delta S}{\delta \eta_\mu^{ab}} + \rho_\mu^{ab} \frac{\delta S}{\delta \vartheta_\mu^{ab}} \right] = 0. \quad (\text{A.10})
 \end{aligned}$$

- The global $U(8)$ invariance

$$\mathcal{Q}_{\mu\nu}^{ab}(S) = 0, \quad (\text{A.11})$$

with

$$\begin{aligned}
 \mathcal{Q}_{\mu\nu}^{ab} \equiv \int d^4x \left(\phi_\mu^{ca} \frac{\delta}{\delta \phi_\nu^{cb}} - \bar{\phi}_\nu^{cb} \frac{\delta}{\delta \bar{\phi}_\mu^{ca}} + \omega_\mu^{ca} \frac{\delta}{\delta \omega_\nu^{cb}} - \bar{\omega}_\nu^{cb} \frac{\delta}{\delta \bar{\omega}_\mu^{ca}} + M_{\sigma\mu}^{ca} \frac{\delta}{\delta M_{\sigma\nu}^{cb}} - \bar{M}_{\sigma\nu}^{cb} \frac{\delta}{\delta \bar{M}_{\sigma\mu}^{ca}} \right. \\
 \left. + N_{\sigma\mu}^{ca} \frac{\delta}{\delta N_{\sigma\nu}^{cb}} - \bar{N}_{\sigma\nu}^{cb} \frac{\delta}{\delta \bar{N}_{\sigma\mu}^{ca}} + \vartheta_\mu^{ca} \frac{\delta}{\delta \vartheta_\nu^{cb}} - \eta_\nu^{cb} \frac{\delta}{\delta \eta_\mu^{ca}} + \rho_\mu^{ca} \frac{\delta}{\delta \rho_\nu^{cb}} - \lambda_{\nu\mu}^{cb} \frac{\delta}{\delta \lambda_\mu^{ca}} \right). \quad (\text{A.12})
 \end{aligned}$$

The presence of this global invariance $U(8)$ allows us to make use of a composite index $i \equiv (a, \mu)$, with $i = 1, \dots, 8$. Thus, from now on, we set

$$(\phi_\mu^{ab}, \bar{\phi}_\mu^{ab}, \omega_\mu^{ab}, \bar{\omega}_\mu^{ab}) = (\phi_i^a, \bar{\phi}_i^a, \omega_i^a, \bar{\omega}_i^a), \quad (\text{A.13})$$

$$(M_{\mu\nu}^{ab}, \bar{M}_{\mu\nu}^{ab}, N_{\mu\nu}^{ab}, \bar{N}_{\mu\nu}^{ab}) = (M_{\mu i}^a, \bar{M}_{\mu i}^a, N_{\mu i}^a, \bar{N}_{\mu i}^a), \quad (\text{A.14})$$

$$(\vartheta_\mu^{ab}, \eta_\mu^{ab}, \rho_\mu^{ab}, \lambda_\mu^{ab}) = (\vartheta_i^a, \eta_i^a, \rho_i^a, \lambda_i^a). \quad (\text{A.15})$$

The trace of (A.12) defines a new Q_8 -charge whose nonvanishing values are displayed in table A1.

- Symmetries involving the Faddeev–Popov ghost fields and the localizing fields

$$\mathcal{W}_i^{(1)}(S) = \int d^4x \left[\phi_i^a \frac{\delta S}{\delta \bar{c}^a} + c^a \frac{\delta S}{\delta \bar{\phi}_i^a} + M_{\mu i}^a \frac{\delta S}{\delta \Omega_\mu^a} - \vartheta_i^a \frac{\delta S}{\delta L^a} + \frac{\delta S}{\delta \lambda_i^a} \frac{\delta S}{\delta b^a} \right] = 0, \quad (\text{A.16})$$

$$\mathcal{W}_i^{(2)}(S) = \int d^4x \left[\bar{\omega}_i^a \frac{\delta S}{\delta \bar{c}^a} - c^a \frac{\delta S}{\delta \omega_i^a} - \bar{N}_{\mu i}^a \frac{\delta S}{\delta \Omega_\mu^a} + \eta_i^a \frac{\delta S}{\delta L^a} + \frac{\delta S}{\delta \rho_i^a} \frac{\delta S}{\delta b^a} \right] = 0, \quad (\text{A.17})$$

$$\begin{aligned} \mathcal{W}_i^{(3)}(S) = \int d^4x \left[\left(\frac{\delta S}{\delta \lambda_i^a} + \omega_i^a \right) \frac{\delta S}{\delta \bar{c}^a} + \frac{\delta S}{\delta \eta_i^a} \frac{\delta S}{\delta b^a} + \left(\frac{\delta S}{\delta \bar{\phi}_i^a} - \rho_i^a \right) \frac{\delta S}{\delta L^a} \right. \\ \left. + c^a \frac{\delta S}{\delta \bar{\omega}_i^a} - M_{\mu i}^a \frac{\delta S}{\delta \xi_\mu^a} + N_{\mu i}^a \frac{\delta S}{\delta \Omega_\mu^a} \right] = 0, \end{aligned} \quad (\text{A.18})$$

$$\begin{aligned} \mathcal{W}_i^{(4)}(S) = \int d^4x \left[\left(\frac{\delta S}{\delta \rho_i^a} - \bar{\phi}_i^a \right) \frac{\delta S}{\delta \bar{c}^a} + \frac{\delta S}{\delta \vartheta_i^a} \frac{\delta S}{\delta b^a} + \left(\frac{\delta S}{\delta \omega_i^a} - \lambda_v^{ab} \right) \frac{\delta S}{\delta L^a} \right. \\ \left. - c^a \frac{\delta S}{\delta \phi_i^a} + \bar{N}_{\mu i}^a \frac{\delta S}{\delta \xi_\mu^a} - \bar{M}_{\mu i}^a \frac{\delta S}{\delta \Omega_\mu^a} \right] = 0. \end{aligned} \quad (\text{A.19})$$

- The exact rigid symmetries associated with the horizon function

$$R_{ij}^{(1)}(S) = \int d^4x \left(\phi_i^a \frac{\delta S}{\delta \omega_j^a} - \bar{\omega}_j^a \frac{\delta S}{\delta \bar{\phi}_i^a} + M_{\mu i}^a \frac{\delta S}{\delta N_{\mu j}^a} + \bar{N}_{\mu j}^a \frac{\delta S}{\delta \bar{M}_{\mu i}^a} + \vartheta_i^a \frac{\delta S}{\delta \rho_j^a} - \eta_j^a \frac{\delta S}{\delta \lambda_i^a} \right) = 0, \quad (\text{A.20})$$

$$R^{(2)}(S) = \int d^4x \left(\bar{\omega}_i^a \frac{\delta S}{\delta \omega_i^a} - \bar{N}_{\mu i}^a \frac{\delta S}{\delta N_{\mu i}^a} - \eta_i^a \frac{\delta S}{\delta \rho_i^a} \right) = 0, \quad (\text{A.21})$$

$$R^{(3)}(S) = \int d^4x \left(\bar{\omega}_i^a \frac{\delta S}{\delta \phi_i^a} - \bar{\phi}_i^a \frac{\delta S}{\delta \omega_i^a} - \bar{N}_{\mu i}^a \frac{\delta S}{\delta M_{\mu i}^a} - \bar{M}_{\mu i}^a \frac{\delta S}{\delta N_{\mu i}^a} - \eta_i^a \frac{\delta S}{\delta \vartheta_i^a} + \lambda_i^a \frac{\delta S}{\delta \rho_i^a} \right) = 0. \quad (\text{A.22})$$

- The global $U(6)$ invariance

$$\mathcal{Q}_{IJ}(S) = 0, \quad (\text{A.23})$$

with the operator \mathcal{Q}_{IJ} given by (61).

- The exact rigid symmetries associated with the mass operator

$$\mathcal{R}_{IJ}^{(\mathcal{N})}(S) = 0, \quad (\text{A.24})$$

with the same operators $\mathcal{R}_{IJ}^{(\mathcal{N})}$, $\mathcal{N} = 1, 2, 3, 4$, already defined in (63).

- The diagonal $U(1)$ Ward identity

$$\mathcal{W}^3(S) = -\partial^2 b, \quad (\text{A.25})$$

with

$$\begin{aligned} \mathcal{W}^3 \equiv \partial_\mu \frac{\delta}{\delta A_\mu} + g \varepsilon^{ab} \left(A_\mu^a \frac{\delta}{\delta A_\mu^b} + b^a \frac{\delta}{\delta b^b} + c^a \frac{\delta}{\delta c^b} + \bar{c}^a \frac{\delta}{\delta \bar{c}^b} + \phi_i^a \frac{\delta}{\delta \phi_i^b} + \bar{\phi}_i^a \frac{\delta}{\delta \bar{\phi}_i^b} \right. \\ + \omega_i^a \frac{\delta}{\delta \omega_i^b} + \bar{\omega}_i^a \frac{\delta}{\delta \bar{\omega}_i^b} + \Omega_\mu^a \frac{\delta}{\delta \Omega_\mu^b} + \tau_\mu^a \frac{\delta}{\delta \tau_\mu^b} + \xi_\mu^a \frac{\delta}{\delta \xi_\mu^b} + M_{\mu i}^a \frac{\delta}{\delta M_{\mu i}^b} + \bar{M}_{\mu i}^a \frac{\delta}{\delta \bar{M}_{\mu i}^b} \\ + N_{\mu i}^a \frac{\delta}{\delta N_{\mu i}^b} + \bar{N}_{\mu i}^a \frac{\delta}{\delta \bar{N}_{\mu i}^b} + \eta_i^a \frac{\delta}{\delta \eta_i^b} + \vartheta_i^a \frac{\delta}{\delta \vartheta_i^b} + \lambda_i^a \frac{\delta}{\delta \lambda_i^b} + \rho_i^a \frac{\delta}{\delta \rho_i^b} + L^a \frac{\delta}{\delta L^b} \\ \left. + B_I^a \frac{\delta}{\delta B_I^b} + \bar{B}_I^a \frac{\delta}{\delta \bar{B}_I^b} + G_I^a \frac{\delta}{\delta G_I^b} + \bar{G}_I^a \frac{\delta}{\delta \bar{G}_I^b} + Y_I^a \frac{\delta}{\delta Y_I^b} + \bar{Y}_I^a \frac{\delta}{\delta \bar{Y}_I^b} + X_I^a \frac{\delta}{\delta X_I^b} + \bar{X}_I^a \frac{\delta}{\delta \bar{X}_I^b} \right). \end{aligned} \quad (\text{A.26})$$

- The off-diagonal $SL(2, \mathbb{R})$ identity

$$\mathcal{D}(S) = \int d^4x \left(c^a \frac{\delta S}{\delta \bar{c}^a} + \frac{\delta S}{\delta L^a} \frac{\delta S}{\delta b^a} \right) = 0. \quad (\text{A.27})$$

- The diagonal gauge fixing

$$\frac{\delta S}{\delta b} = \partial_\mu A_\mu. \quad (\text{A.28})$$

- The diagonal anti-ghost equation

$$\frac{\delta S}{\delta \bar{c}} + \partial_\mu \frac{\delta S}{\delta \Omega_\mu} = 0. \quad (\text{A.29})$$

We note here that no diagonal ghost equation, similar to (48), holds when the horizon function is taken into account [38]. However, the set of Ward identities listed above forbids the presence of counterterms such as

$$\int d^4x (\bar{\phi}_i^a \phi_i^a - \bar{\omega}_i^a \omega_i^a) (\bar{B}_I^A B_I^A - \bar{G}_I^A G_I^A), \quad (\text{A.30})$$

as well as of any other counterterm which would mix fields associated with the two different nonlocal operators (5) and (A.1). Therefore, in complete analogy with the case of the Landau gauge [53], the two operators (5) and (A.1) do not mix, due to the rich symmetry content of the resulting local action. Moreover, it turns out that the most general allowed counterterm can be in fact reabsorbed in the starting action (A.7) through a redefinition of fields, parameters and sources, ensuring the renormalizability of the mass operator (5) in the presence of the horizon function (A.1).

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